BRIEF COMMUNICATIONS

THE APPLICATION OF TRANSFER FUNCTIONS TO THE ANALYSIS OF THE DYNAMICS OF THE INITIAL HEATING OF STEAM TURBINES

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By analyzing the frequency characteristics of the initial heating of a steam turbine casing wall we show that the application of approximate expressions for the transfer function gives a description of the change in the temperature differences at the wall which is qualitatively and quantitatively incorrect. In analyzing the dynamics of initial heating for the purposes of automatic control, we must take into account the initial thermal state of the wall.

In constructing algorithms for the automatic control of turbine start-up we have to know the dynamic characteristics of their initial heating, the dynamics of the change in the factors determining whether changes can be made in the start-up procedures. One of these factors is frequently taken to be the rate of initial heating of the turbine casing wall which is considered as an index of the level of the temperature stresses in the wall. It is more correct to take the temperature differences in the wall as such an index.

To simulate and analyze the initial heating dynamics of steam turbine casings in connection with problems in the automatic control of turbine start-up a number of authors use the method of transfer functions [1-4]. For a turbine casing wall, simulated as an unbounded flat plate, the transfer function with parameter s has the form

$$Y(s)=\frac{\operatorname{ch} x\sqrt{Ts}}{\operatorname{ch}\sqrt{Ts}+\frac{\sqrt{Ts}}{\mathrm{Bi}}\operatorname{sh}\sqrt{Ts}}. \tag{1}$$

By expanding the expressions in the numerator and denominator in power series we can replace (1) by a rational function. In [1-4] the elements of the casing were taken as second and first order inertial links to give a dynamic approximation model

$$Y(s)\approx\frac{1+b_1 s}{1+a_1 s+a_2 s^2}. \tag{2}$$

Equation (2), which is a second approximation to (1), is general for bodies of simple geometrical shape when there is heat exchange between the body and the surrounding medium [5, 6]. The coefficients a_1, a_2, b_1 are functions of the shape of the body, the thermophysical properties, and the conditions under which heat exchange takes place. For an unbounded flat plate

$$b_1=\frac{x^2}{2a};\quad a_1=\frac{R^2}{a}\frac{2+\mathrm{Bi}}{2\mathrm{Bi}};\quad a_2=\frac{R^4}{6a^2}\frac{4+\mathrm{Bi}}{4\mathrm{Bi}}.$$

The accuracy that may be achieved by the approximation can be verified by comparing the exact frequency characteristics of initial heating with those obtained from (2). The amplitude and phase shift for linear and linearized systems can be determined if we know the transfer function of the system by replacing the parameter s in it by $i\omega$ where ω is the cyclic frequency of the oscillations. Then the amplitude–frequency characteristic is defined as $A(\omega) = \mathrm{mod}\,Y(i\omega)$ and the phase–frequency characteristic as $\varphi(\omega) = \arg Y(i\omega)$. Thus, from (2), we obtain:

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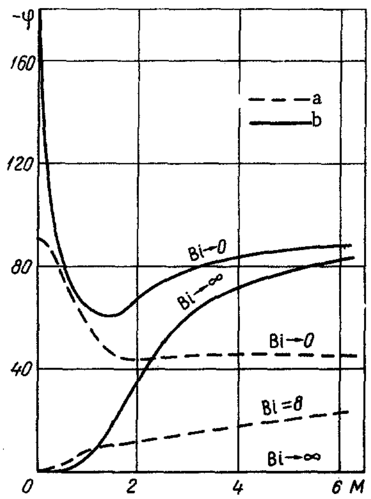

Fig. 1

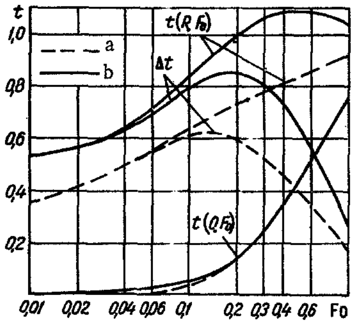

Fig. 2

Fig. 1. Phase–frequency characteristics for the heated surface of the plate: a) the exact solution; b) the approximate solution.

Fig. 2. Initial heating of an unbounded plate for unit change in the steam temperature: a) the exact solution $t(x, \mathrm{Fo}) = 1 - \Sigma A_n \cos m_n \cdot (x/R) \exp(-m_n^2 \mathrm{Fo})$; b) the solution obtained from the second approximation for the transfer function (2).

$$A(\omega) = \frac{\{1 + (a_1^2 + b_1^2 - 2a_2)\omega^2 + [b_1^2(a_1^2 - 2a_2) + a_2^2]\omega^4 + a_2^2 b_1^2 \omega^6\}^{\frac{1}{2}}}{1 + (a_1^2 - 2a_2)\omega^2 + a_2^2\omega^4}, \tag{3}$$

$$\varphi(\omega) = \operatorname{arc\,tg}\left[\omega \frac{b_1 - a_1 - a_2 b_1 \omega^2}{1 + (a_1 b_1 - a_2)\omega^2}\right]. \tag{4}$$

From the exact solution of the Fourier equation when the temperature of the medium is subject to simple harmonic oscillations [7] we obtained the following expressions for the frequency characteristics with boundary conditions of the third kind, after various transformations:

$$A = \left\{\frac{\mathrm{Cc}^2 M \frac{x}{R} + \mathrm{Ss}^2 M \frac{x}{R}}{\left[\mathrm{Cc}\, M + \frac{M}{\mathrm{Bi}}(\mathrm{Sc}\, M - \mathrm{Cs}\, M)\right]^2 + \left[\mathrm{Ss}\, M + \frac{M}{\mathrm{Bi}}(\mathrm{Sc}\, M + \mathrm{Cs}\, M)\right]^2}\right\}^{\frac{1}{2}}, \tag{5}$$

$$\varphi = \operatorname{arc\,tg}\left\{\left[\mathrm{Cc}\, M\, \mathrm{Ss}\, M \frac{x}{R} - \mathrm{Ss}\, M\, \mathrm{Cc}\, M \frac{x}{R}\right] + \frac{M}{\mathrm{Bi}}\left[\mathrm{Ss}\, M \frac{x}{R}(\mathrm{Sc}\, M - \mathrm{Cs}\, M) - \mathrm{Cc}\, M \frac{x}{R}(\mathrm{Sc}\, M + \mathrm{Cs}\, M)\right]\right\}$$
$$\times \left\{\left[\mathrm{Cc}\, M\, \mathrm{Cc}\, M \frac{x}{R} + \mathrm{Ss}\, M\, \mathrm{Ss}\, M \frac{x}{R}\right] + \frac{M}{\mathrm{Bi}}\left[\mathrm{Ss}\, M \frac{x}{R}(\mathrm{Sc}\, M + \mathrm{Cs}\, M) + \mathrm{Cc}\, M \frac{x}{R}(\mathrm{Sc}\, M - \mathrm{Cs}\, M)\right]\right\}^{-1}. \tag{6}$$

For boundary conditions of the first kind ($\mathrm{Bi} \to \infty$), expressions (5), (6) become the familiar Greber expressions [8]. Equations (5), (6) also simplify considerably for boundary values of x.

Comparison of the frequency characteristics in (3)-(6) confirms that it is permissible to represent the transfer function as the second approximation (2) in analyzing the temperature changes at the outer isolated surface of the wall. To analyze the changes at the inner heated surface a higher order approximation is necessary. Figure 1 shows the phase–frequency characteristics calculated from (4), (6) for the heated surface and limiting values of the Biot criterion. By comparing the graphs we see that when we use an approximate expression for the transfer function to analyze the thermal processes a qualitatively incorrect picture is obtained. Figure 2 shows curves for the changes in the metal temperatures on the boundary surfaces and the temperature difference in the wall for a particular case (discontinuous unit change in the steam temperature, homogeneous initial conditions, Bi = 4), obtained using (2) and the exact solution [7]. Comparison of the curves confirms the conclusion we have reached.

In using transfer functions the initial temperature at the wall is assumed to be uniform and the initial heating process is assumed to be regular.

If we use the time principle to construct algorithms for start-up control the effect of the initial time interval to the onset of regular conditions of heating can be ignored because it is small in comparison with the total start-up time. In constructing algorithms for control by the control index we have to take into account the effect of the initial temperature field to predict its changes [9].

NOTATION

x	is the distance from the isolated surface of the wall;
R	is the wall thickness;
a	is the thermal conductivity coefficient;
$T = R^2/a$	is the time constant;
Bi	is the Biot criterion;
$\mathrm{Ccz} = \mathrm{ch}z \cos z$;	
$\mathrm{Ssz} = \mathrm{sh}z \sin z$;	
$\mathrm{Csz} = \mathrm{ch}z \sin z$;	
$\mathrm{Scz} = \mathrm{sh}z \cos z$.	

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